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AN ESTIMATOR OF FINITE POPULATION VARIANCE

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SUMMARY

This paper proposes a class of estimators of population variance more efficient than the one considered by Srivastava and Jhajj [1].

Keywords : Auxiliary information, Simple random sampling, Ratio-type estimators, Asymptotic mean square error.

Introduction and Notation

Srivastava and Jhajj [1] defined a class of estimators of population variance σ_y^2. For the case of single auxiliary variable, the class considered by Srivastava and Jhajj [1] is

$$t_h = s_y^2 \, h(u, v)$$

where $u = s_x^2/S_x^2$, $v = \bar{x}/\bar{X}$ and $h(.,.)$ is a parametric function satisfying some usual regularity conditions. The mean squared error (up to terms of order, n^{-1}) of t_h is

$$\text{MSE}(t_h) = n^{-1}\sigma_y^4\left[\{\beta^2(y) - 1\} - \frac{(l-1)^2 C_x^2 + K^2\{\beta_2(x) - 1\} - 2(l-1)KM}{\{\beta_2(x) - 1\}C_x^2 - M^2}\right]$$

2. The Class of Estimators

We have proposed a class of estimators as

$$t_s = s_y^2\, h(u, v) + \frac{\alpha}{s_y^2} \tag{2.1}$$

where $u = s_x^2/S_x^2$, $v = \bar{x}/\bar{X}$. Whatever be the sample chosen $h(u, v)$ assum- the valve in a bounded closed convex sub-set R_2 of two dimentional l space containing the point (1, 1) s.t. $h(1, 1) = 1$.

ppose a SRSWR of size n is drawn from population of size N. Let n, so that f.p.c. terms can be ignored throughout. Write

$$\varepsilon = (s_y^2/S_y^2) - 1, \quad \eta = u - 1, \quad \delta = v - 1$$

$$E(\epsilon) = E(\eta) = E(\delta) = 0$$

nd

$$E(\epsilon^2) = n^{-1}\{\beta_2(y) - 1\}, \quad E(\eta^2) = n^{-1}\{\beta_2(x) - 1\}$$

$$E(\delta^2) = n^{-1} C_x^2, \quad E(\varepsilon\,\eta) = n^{-1}(l - 1),$$

$$E(\epsilon\delta) = n^{-1} K, \quad E(\eta\,\delta) = n^{-1} M$$

where

$$K = u_{21}(y, x)/\bar{X}\,\sigma_y^2, \quad M = u_3(\bar{x})/\bar{X}\;\sigma_x^2, \; l = u_{22}(y, x)\,\sigma_x^2\,\sigma_y^2$$

$$\bar{x} = n^{-1}\sum_{i=1}^{n} x_i, \quad s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Using second order Taylor's series, t_s in terms of ε, η and δ about the point (1, 1) can be written as

$$t_s = \sigma_y^2\,[1 + h_1(1, 1)\,\delta + h_2(1, 1)\,\eta + \epsilon + h_1(1, 1)\,\epsilon\,\delta + h_2(1, 1)\,\epsilon\,\eta\,] + \frac{\alpha}{\sigma_y^2}(1 - \epsilon + \epsilon^2 + \cdots) \tag{2.2}$$

where $h_1(1, 1)$ and $h_2(1, 1)$ denote the first order partial derivatives of $h(u, v)$ w.r.t. u and v respectively. Clearly the bias of the proposed class of estimators is of order n^{-1}. The mean squared error (upto terms of order n^{-1}) is given by

$$\text{MSE}(t_s) = n^{-1}\sigma_y^4 [\{\beta_2(y) - 1\} + h_1^2(1,1)C_x^2 + h_2^2(1,1)\{\beta_2(x) - 1\} + 2h_1(1,1)K + 2h_2(1,1)(l-1) + 2h_1(1,1)h_2(1,1)M] + \frac{\alpha^2}{\sigma_y^4}[1 + 3n^{-1}\{\beta_2(y) - 1\}] - 2\alpha n^{-1}\{\beta_2(y) - 1\} \quad (2.3)$$

Mean squared error is minimised for

$$h_1(1,1) = -\frac{K\{\beta_2(x) - 1\} - (l-1)M}{\{\beta_2(x) - 1\}C_x^2 - M^2}$$

$$h_2(1,1) = -\frac{C_x^2(l-1) - KM}{\{\beta_2(x) - 1\}C_x^2 - M^2} \quad \text{and}$$

$$\alpha = \frac{n^{-1}\sigma_y^4\{\beta_2(y) - 1\}}{1 + 3n^{-1}\{\beta_2(y) - 1\}}$$

Hence, the resulting (minimum) mean squared error (up to terms of order n^{-1}) is given by

$$\min M(t_s) = \text{MSE}(t_h) - \frac{n^{-2}\sigma_y^4\{\beta_2(y) - 1\}}{1 + 3n^{-1}\{\beta_2(y) - 1\}}$$

Obviously, the proposed class of estimators is more efficient than the class of estimators considered by Srivastava and Jhajj [1].

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REFERENCE

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